

Thermodynamics of n - p condensate in asymmetric nuclear matter

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We study the neutron-proton pairing in nuclear matter as a function of isospin asymmetry at finite temperatures and the saturation density using realistic nuclear forces and Brueckner-renormalized single particle spectra. Our computation of the thermodynamic quantities shows that while the difference of the entropies of the superconducting and normal phases anomalously changes its sign as a function of temperature for arbitrary asymmetry, the grand canonical potential does not; the superconducting state is found to be stable in the whole temperature-asymmetry plane. The pairing gap completely disappears for density-asymmetries exceeding $\alpha_c = (n_n - n_p)/n \simeq 0.11$.

Neutron-proton pair correlations are potentially important in a number of contexts, including the study of the nuclear structure of medium mass $N \simeq Z$ nuclei produced at the radioactive nuclear beam facilities [1] and the theory of the deuteron formation in the medium energy heavy-ion collisions [2]. In the astrophysical context n - p pairing correlations are relevant for the astrophysical r -process [3,4] and could play a major role in neutron star models which permit pion or kaon condensation [5].

In recent years much theoretical effort has been devoted to the understanding of the n - p pair correlations. They have been studied both in the infinite nuclear matter within the Thouless [6,7] or the Bardeen-Cooper-Schrieffer [8] theories of superconductivity [9–16], and in finite nuclei within mean-field effective interaction theories of pairing [1,17–23]. In particular, microscopic calculations, based on the BCS theory for the bulk nuclear matter show that the isospin-asymmetric matter supports Cooper-type pair correlations in the 3S_1 - 3D_1 partial-wave channel due to the tensor component of the nuclear force. The energy gap is of the order of 10 MeV at the saturation density [9–11,16] when the effects of the medium polarization on the pairing force [24–28] are neglected.

As the existence of the pair correlations crucially depends upon the overlap between the neutron and proton Fermi surfaces, one expects a suppression of the pairing correlations if the system is driven out of the isospin-symmetric state. The studies based on the Thouless criterion [6,7] for the thermodynamic T -matrix (the divergence of the ladder resummation scheme at the critical temperature) deduced the suppression of the critical temperature with the isospin asymmetry [12,13]; however, they do not permit to draw conclusions about properties of the superconducting state with a finite gap. The studies based on the BCS theory without self-energy-corrections to the single particle spectrum [14,15], while give a correct qualitative picture, overestimate the magnitude of the pairing gap and the critical asymmetries at which the pairing disappears.

In this paper we report on a fully microscopic calculation of the n - p pairing within the strong coupling theory

of the superconductivity with realistic nuclear interactions. Our main focus is the effect of ladder-renormalized single particle energies [29] on the magnitude of the pairing gap and the threshold asymmetries at which the pairing vanishes [30]. Our second aim is to provide a first computation of the key thermodynamic quantities of asymmetric n - p condensate from the strong coupling BCS theory.

To set the stage, let us start with the solution of the Dyson equations for the normal and anomalous propagators in the Matsubara formalism. In doing so we shall decouple isospin singlet SD pairing channel from the isospin triple channels, as the SD coupled channels contain the dominant part of the attractive pairing force. In this case the pairing matrix is diagonal in the spin-space, i.e. one deals with the unitary triplet state (see ref. [11]). The proton/neutron propagators follow from the solution of the Gor'kov equations, and can be cast in the form ($\hbar = 1$)

$$G_{\sigma,\sigma'}^{(p/n)}(\vec{k}, \omega_m) = -\delta_{\sigma,\sigma'} \frac{i\omega_m + \xi_k \mp \delta\varepsilon_{\vec{k}}}{(i\omega_m + E_k^+)(i\omega_m - E_k^-)}; \quad (1)$$

the neutron-proton anomalous propagator has the form

$$F_{\sigma,\sigma'}^\dagger(\vec{k}, \omega_m) = -\delta_{\sigma,\sigma'} \frac{\Delta^\dagger(\vec{k})}{(i\omega_m + E_k^+)(i\omega_m - E_k^-)}, \quad (2)$$

where ω_m are the Matsubara frequencies, the upper sign in $G^{(p/n)}$ corresponds to protons and the lower to neutrons. The isospin asymmetry lifts the degeneracy of the quasiparticle spectra thus leading to two separate branches for protons and neutrons

$$E_k^\pm = \sqrt{\xi_k^2 + \Delta_k^2} \pm \delta\varepsilon_{\vec{k}}, \quad (3)$$

where

$$\xi_{\vec{k}} \equiv \frac{1}{2} \left(\varepsilon_{\vec{k}}^{(n)} + \varepsilon_{\vec{k}}^{(p)} \right), \quad \delta\varepsilon_{\vec{k}} \equiv \frac{1}{2} \left(\varepsilon_{\vec{k}}^{(p)} - \varepsilon_{\vec{k}}^{(n)} \right),$$

and $\varepsilon_{\vec{k}}^{(n,p)}$ are the single particle energies of neutrons and protons. The strong coupling BCS theory is coupled to

the Brueckner renormalization scheme via the single particle energies defined as $\varepsilon_{\vec{k}}^{(n,p)} = k^2/2m + U^{(n,p)}(k) - \mu^{(n,p)}$; here $U^{(n,p)}(k)$ are the single particle potentials which are derived from the Brueckner theory of asymmetric nuclear matter [29] and $\mu^{(n,p)}$ are the chemical potentials for neutrons and protons, which are derived from the BCS theory self-consistently. Both schemes are normalized to the same densities. The small effect of the feedback of the pairing correlation in the Brueckner calculations of the mean-field [31] is, however, neglected.

Using the angle-averaging procedure, which is an adequate approximation for the present purpose (see ref. [11]) the BCS gap equation for asymmetric nuclear matter can be derived. We find

$$\Delta_l(k) = - \sum_{k'} \sum_{l'} V_{ll'}(k, k') \times \frac{\Delta_{l'}(k')}{2\sqrt{\xi_{k'}^2 + D(k')^2}} [1 - f(E_{\vec{k}}^+) - f(E_{\vec{k}}^-)], \quad (4)$$

where $D(k)^2 \equiv \Delta_0(k)^2 + \Delta_2(k)^2$ is the angle-averaged neutron-proton gap function, $f(E) = [1 + \exp(\beta E)]^{-1}$ is the Fermi distribution function, $\beta^{-1} = k_B T$, where T is the temperature and k_B is the Boltzmann constant. The driving term, $V_{ll'}$, is the bare interaction in the SD channel. The density matrices of neutrons and protons follow from eq. (1) after summation over frequencies,

$$n_{\sigma}^{(p/n)}(k) = \left\{ \frac{1}{2} \left(1 + \frac{\xi_{\vec{k}}}{\sqrt{\xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2}} \right) f(E_{\vec{k}}^{\pm}) + \frac{1}{2} \left(1 - \frac{\xi_{\vec{k}}}{\sqrt{\xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2}} \right) [1 - f(E_{\vec{k}}^{\mp})] \right\}. \quad (5)$$

Summation over frequencies in eq. (2) leads to the density matrix of the particles in the condensate

$$\nu(\vec{k}) = \frac{\Delta(\vec{k})}{2\sqrt{\xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2}} [1 - f(E_{\vec{k}}^+) - f(E_{\vec{k}}^-)]. \quad (6)$$

The partial densities of nucleons, in terms of the density matrices, are $\rho^{(p/n)} = \sum_{\vec{k}, \sigma} n_{\sigma}^{(p/n)}(k)$; similarly, $\rho^{\text{cond}} = \sum_{\vec{k}, \sigma} \nu_{\sigma}(k)$ is the density of the particles within the condensate. It is essential that the coupled system of equations (4) and (5) are solved self-consistently.

Now we are in the position to write down the key thermodynamic quantities. As the occupation of the quasi-particle states is given by the Fermi-Dirac distribution function, the entropy of the system is given by the combinatorial expression

$$S = -2k_B \sum_{\vec{k}} \left\{ f(E_{\vec{k}}^+) \ln f(E_{\vec{k}}^+) + \bar{f}(E_{\vec{k}}^+) \ln \bar{f}(E_{\vec{k}}^+) + f(E_{\vec{k}}^-) \ln f(E_{\vec{k}}^-) + \bar{f}(E_{\vec{k}}^-) \ln \bar{f}(E_{\vec{k}}^-) \right\}, \quad (7)$$

where $\bar{f}(E_{\vec{k}}^{\pm}) = [1 - f(E_{\vec{k}}^{\pm})]$. Equation (7) is an obvious extension of the mean-field expression for the entropy in the symmetrical nuclear matter [11] to the asymmetric case. The internal energy, defined as the grand canonical statistical average of the Hamiltonian, $\mathcal{U} = \langle \hat{H} - \mu^{(n)} \hat{\rho}_n - \mu^{(p)} \hat{\rho}_p \rangle$, reads

$$\mathcal{U}(T, \mu) = \sum_{\sigma \vec{k}} \left[\varepsilon_{\vec{k}}^{(n)} n_{\sigma}^{(n)}(k) + \varepsilon_{\vec{k}}^{(p)} n_{\sigma}^{(p)}(k) \right] + \sum_{\vec{k}_1 \vec{k}_2} V(\vec{k}_1, -\vec{k}_1; \vec{k}_2, -\vec{k}_2) \nu(\vec{k}_1) \nu(\vec{k}_2), \quad (8)$$

where we have carried out the spin summation in the second term. The non-pairing interaction energy among the quasiparticles and the contribution from the chemical potentials are included in the single particle energies (the first term) of eq. (8). The second term includes the BCS mean field interaction among the particles in the condensate. Note that the interaction in the second term of eq. (8) can be eliminated in terms of the gap equation (4). Finally, the thermodynamic potential is given as

$$\Omega(T, \mu) = \mathcal{U}(T, \mu) - TS. \quad (9)$$

The expressions for the entropy and the internal energy in the normal state follow from eqs. (7) and (8) in the limit $\Delta = 0$. Our main interest below is the change in the thermodynamic potential of the superconducting state with respect to the normal state $\delta\Omega = \Omega_s - \Omega_n$.

Numerical calculations of the pairing gap were carried out using the Paris potential. The single particle potentials were used as an input from Brueckner computation based on the Argonne V14 potential; for details of these calculations we refer to ref. [29].

Fig. 1 shows the values of the pairing gap in the 3S_1 - 3D_1 partial wave channel as a function of the temperature at the saturation density $\rho = 0.17 \text{ fm}^{-3}$. The asymmetry parameter is defined as $\alpha \equiv (\rho_n - \rho_p)/\rho$. For the value $\alpha = 0$ one finds the usual BCS solution: the gap is a monotonically decreasing function of the temperature and vanishes at the critical temperature $T_c = \Delta(T=0)/1.76$. In the asymmetric state the gap develops a maximum at a certain intermediate temperature in the range $0 < T < T_c$; the value of the critical temperature is reduced and the BCS relation between the critical temperature and the value of the gap at $T = 0$ does not hold any longer. For large asymmetries $\alpha \sim 0.1$ the superconducting state exists only in a finite temperature range; it completely vanishes for the critical asymmetry $\alpha_c = 0.11$. This behavior is the result of the interplay between the increasing shift between the radii of the Fermi spheres of neutrons and protons with increasing asymmetry and the smearing of the Fermi surfaces due to *both* interaction driven correlations and the temperature. While the first factor tends to suppress the pairing, the role of the second factor is twofold. On the one hand the temperature and correlations smear the Fermi surfaces thus increasing the overlap between the two Fermi

spheres which promotes the pairing. If, however, the temperatures are high enough the pair correlated states are quenched by the thermal excitation. The pairing gap, hence, has a maximum at some intermediate temperature. The two critical temperatures are controlled by two different mechanisms. The superconductivity vanishes with decreasing temperature at a lower critical temperature when the smearing becomes insufficient to support the pairing. The superconductivity vanishes at the upper critical temperature because of the thermal excitation of the system, as in the standard BCS theory.

Qualitatively, the role of the mean-field can be understood in terms of the effective mass. As in the symmetric matter the nucleon effective mass, m^* , is reduced by the mean-field, the density of states at the Fermi surface is reduced, and therefore the pairing gap is reduced as well. In asymmetric matter there are no additional non-trivial effects. Even though the neutron and proton effective masses change with the density in opposite directions, the gap itself depends on the reduced effective mass $(m_p^* + m_n^*)/2m_p^*m_n^*$, which scales with the asymmetry as $1/m^* + \mathcal{O}(\alpha^2)$.

Fig. 2 displays the entropies of the normal and superconducting states as a function of the temperature at the saturation density. The $\alpha = 0$ behavior of the normal and superconducting entropies is the one expected from the theory of the normal Fermi-liquids and the BCS theory, respectively: the entropy of the normal state is a linear function of the temperature; the entropy of the superconducting state is linear close to the critical temperature and decreases exponentially in the low temperature limit. For finite asymmetries the entropy of the normal state is unchanged. The curve of the entropy of superconducting state, however, shows an anomalous behaviour by crossing the entropy curve of the normal state. For temperatures below the crossing point *the entropy of the superconducting state is larger than that of the normal state*. The crossover to the anomalous behaviour occurs at the temperature at which the gap reaches its maximum. This anomalous dependence of the entropy of the superconducting state on the temperature implies that this state could be unstable for temperatures below the temperature at which the gap attains its maximum. The internal energy, the first term in eq. (9), has a definite sign as it vanishes in the normal state and is a quadratic form of the pairing gap. Thus the sufficient condition for the onset of the instability is that the contribution of the second term in eq. (9) dominates that of the first in the range of the temperatures below the crossover point.

Fig. 3 shows the difference in the thermodynamic potentials of the normal and superconducting state $\delta\Omega$. This quantity is negative for all temperatures and asymmetries. The anomalous change of sign of $S_n - S_s$, therefore, does not change the net balance between the energies of the normal and superconducting states. The temperature dependence of $\delta\Omega$ is the reminiscent of the temperature dependence of the gap function by virtue of

the (approximately) quadratic dependence of the internal energy on the pairing gap. We thus conclude that superconducting state is stable, at least in the present model, in the whole temperature-asymmetry plane, whenever a non-trivial solution to the gap equation exists. We have verified that the deduced behaviour of the $\delta\Omega$ is the consequence of the fact that the contribution of the internal energy to the thermodynamic potential dominates the contribution from the entropy.

The problem of the pairing suppression and the thermodynamics of the superconducting phase when the particles lie on different Fermi-surfaces is encountered in many systems. The simplest example is a superconducting metal in a magnetic field, where the spin degeneracy is relaxed because of the Pauli paramagnetism [32–34]. The spin zero pairing between the spin-up and spin-down populations is precisely the one described above, albeit one commonly uses the BCS weak coupling ansatz which does not work for nuclear matter. Another example is the B-state of liquid ^3He (Balian-Werthamer state) in the magnetic field [35]. In both cases measurements of the specific heat and spin susceptibility could provide an experimental information on the anomalous behaviour of the entropy. Apart from condensed matter, our results may be relevant for the current discussion of spontaneous breaking of color symmetry in finite temperature/density QCD and formation of $\langle qq \rangle$ color superconducting condensate [36–39]. In all cases, it would be interesting to check whether the balance between the internal energy and the entropy is generically such that the superconducting state remains stable for arbitrary asymmetries.

To conclude, we studied the pairing in the isospin asymmetric nuclear matter using realistic nuclear interactions combined with ladder-renormalized single particle energies. The pairing at the saturation density vanishes for the critical density asymmetry $\alpha_c = 0.11$ at the finite temperature $T \simeq 1.5$ MeV. Our evaluation of the thermodynamic quantities of the isospin asymmetric nuclear matter shows that, while the entropy of the superconducting state becomes larger than that of the normal state below a certain temperature, the thermodynamical potential at the saturation density corresponds to a stable superconducting state in the whole asymmetry-temperature plane.

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FIG. 1. The pairing gap at the saturation density as a function of the temperature for asymmetries $\alpha = 0, 0.05, 0.07$ and 0.1 .

Fig. 1

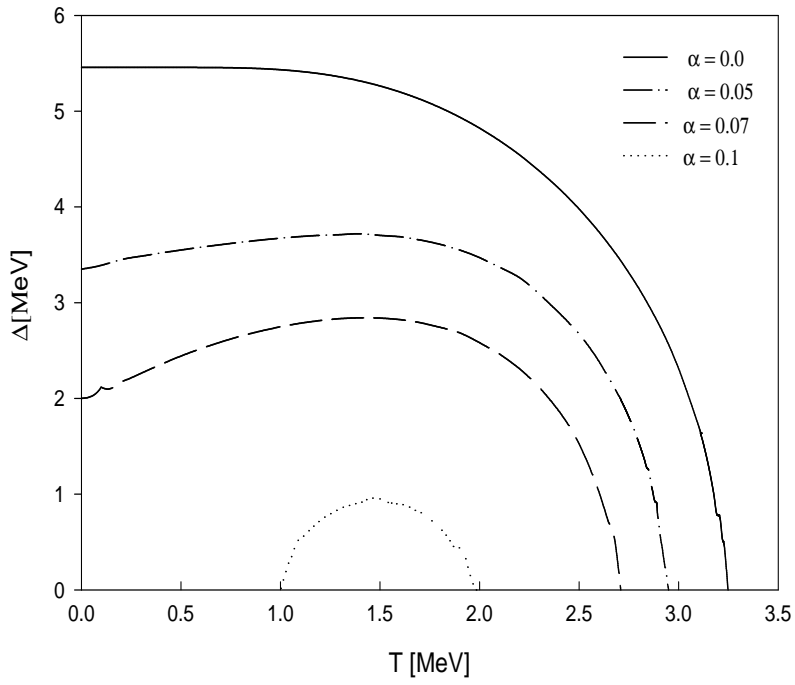


FIG. 2. The entropy of normal (solid line) and superconducting state for $\alpha = 0$ and 0.05, 0.07; the $\alpha = 0.1$ curve is indistinguishable from the normal state entropy curve on figure's scale.

Fig. 2

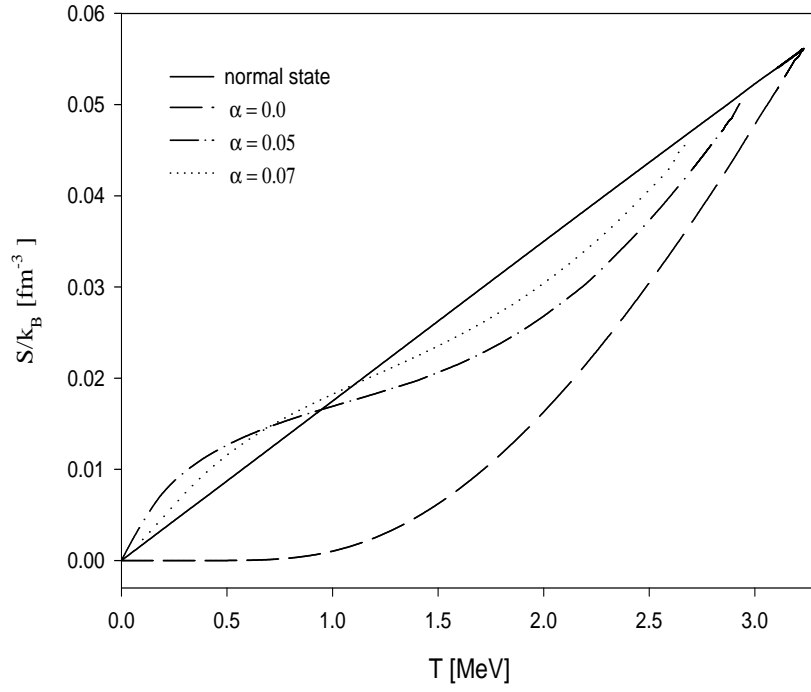


FIG. 3. The difference in the thermodynamic potentials of the superconducting and normal states at the saturation density as a function of the temperature for asymmetries $\alpha = 0, 0.05, 0.07$ and 0.1 .

Fig. 3

